

Name: Solutions

Section: _____

1. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

(a) How many solutions are there to the equation $A\vec{x} = \vec{0}$?

(pivot in both columns)

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 0 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -2 & 0 \\ 0 & -4 & 0 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{array} \right] -$$

there is a unique solution

(b) Let $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. How many solutions are there to the equation $A\vec{x} = \vec{b}$?

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 4 & 1 \\ 5 & 6 & 1 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -2 & -2 \\ 0 & -4 & -4 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{array} \right]$$

there is a unique solution

(pivot in both columns)

(c) Let $\vec{c} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. How many solutions are there to the equation $A\vec{x} = \vec{c}$?

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 4 & 0 \\ 5 & 6 & 1 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -2 & 0 \\ 0 & -4 & -4 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -4 \end{array} \right]$$

there are no solutions

(d) How many solutions are possible to $A\vec{x} = \vec{d}$, when considering all vectors $\vec{d} \in \mathbb{R}^3$?
You must justify your answer.

there either one or none solutions

there is a
pivot in ea.
column

there is not
a pivot in
each row.

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2. Let $A = \begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}$

(a) How many solutions are there to the equation $A\vec{x} = \vec{0}$?

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ -3 & 6 & 0 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

↑
no pivot

there are ∞ -many solutions

(b) Let $\vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. How many solutions are there to the equation $A\vec{x} = \vec{b}$?

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ -3 & 6 & 1 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & 4 \end{array} \right]$$

there are no solutions

(c) Let $\vec{c} = \begin{bmatrix} 5 \\ -15 \end{bmatrix}$. How many solutions are there to the equation $A\vec{x} = \vec{c}$?

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ -3 & 6 & -15 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 0 & 0 \end{array} \right]$$

↑
no pivot

there are ∞ -many slns

(d) How many solutions are possible to $A\vec{x} = \vec{d}$, when considering all vectors $\vec{d} \in \mathbb{R}^2$?
You must justify your answer.

there are either ∞ -many or no solutions

column 2
lacks a pivot

row 2 lacks
a pivot

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3. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

(a) How many solutions are there to the equation $A\vec{x} = \vec{0}$?

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \end{array} \right]$$

there are ∞ -many solutions

↑ No pivot

(b) Let $\vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. How many solutions are there to the equation $A\vec{x} = \vec{b}$?

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -3 \end{array} \right]$$

there are ∞ -many solutions

↑ No pivot

(c) Let $\vec{c} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. How many solutions are there to the equation $A\vec{x} = \vec{c}$?

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 2 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \end{array} \right]$$

↑ No pivot

there are ∞ -many solutions

(d) How many solutions are possible to $A\vec{x} = \vec{d}$, when considering all vectors $\vec{d} \in \mathbb{R}^2$?
You must justify your answer.

there are ALWAYS ∞ -many solutions. ~~always~~

Because ~~no~~ pivot in both rows \Rightarrow always has a sol.
AND no pivot in col 3 \Rightarrow has ∞ -many solns.